of adequate means for its pursuit. By such means will our rulers show themselves to be the real well-wishers and benefactors of their country.

TODHUNTER'S "MATHEMATICAL THEORIES OF ATTRACTION"[*]

A History of the Mathematical Theories of Attraction and the Figure of the Earth from the time of Newton to that of Laplace. By I. Todhunter, M.A., F.R.S. Two vols. (London: Macmillan, 1874.)

II.

Of the great Scotch mathematician, Maclaurin, we read—"The importance of his investigations may be seen by observing how great has been his influence on succeeding writers. Clairaut, D'Alembert, Lagrange, Legendre, Gauss, Ivory, and Chasles show by reference, explicit or implicit, their obligations to the creator of the theory of the attraction of ellipsoids.

Maclaurin well deserves the memorable association of his name with that of the great master in the inscription which records that he was appointed Professor of Mathematics at Edinburgh, "*ipso Newtono suadente*." His main contribution to the theory of the figure of the earth was an exact demonstration of Newton's postulate, of which only approximate solutions had previously been given. We may note on § 260 that we have seen the French version of his "Treatise on Fluxions," "traduit de l'Anglois, par le R. P. Pezenas, Paris, 1749; 2 vols." The first volume has li. pp. of Introduction; v.-viii. *Avertissement par le traducteur* ix.-li., Translation of Author's Preface and Introduction with *Table des Matières*; then 344 pp. of text, plates, and 4 pp. of errata. The second volume has viii. pp. of contents, 322 pp. of text, plates, with 6 pp. at end (4 pp. of errata), for errata, *approbation*, and *privilège*.

The next noteworthy name is again that of an English writer, Thomas Simpson. His contributions are of eminent importance. The analysis he employed, Mr. Todhunter observes, "would not have been unworthy of Laplace himself." There is here an interesting biographical note of a kind which the writer so well knows how to introduce, and which adds a charm to the more general details. In writing our notice we have especially dwelt upon the English contributors to our subject; on the whole it can hardly be denied "that Newton's countrymen have left to foreigners the glory of continuing and extending his empire." Singularly enough Mr. Todhunter gives no account of Simpson's work, "A New Treatise of Fluxions . . . with a variety of new and curious problems." London, 1737. 8vo. This is six years earlier than the "Mathematical Dissertations." Problems XXI. to XXIII. (§§ 201-206) deal with attractions of a circular plate on a point on the axis; of a cylinder on a point on its axis; of a sphere on a particle on its surface, or any distance above it, for law varying as inverse of $(\text{distance})^2$ and for $(\text{distance})^n$. They correspond to Problems II., IV., V., VI. of 1823 edition of the "Doctrine and Application of Fluxions."

The great work of Clairaut, "Théorie de la figure de la terre," &c., appeared in 1743. In this branch "no other person has accomplished so much as Clairaut; and the subject remains at present substantially as he left it, though the form is different. The splendid analysis which Laplace supplied adorned but did not really alter the theory which started from the creative hands of Clairaut." Laplace, too, places it "au rang des plus belles productions mathématiques."

The expedition to Peru gave rise to much paper warfare, and Mr. Todhunter has collected together, in a useful form, the titles of the original pamphlets. We think he has overlooked the following, "Nouveau projet d'une mesure invariable propre à devenir universelle, extrait d'un memoire lû . . . le 24 avril, 1748, par M. de la Condamine," viii. pp. A copy we have consulted of No. xx. (p. 236) is dated "Plombières, juin 30, 1754." (Consult Lalande, p. 455.)

D'Alembert need not long detain us. Laplace points out that his writings want "clarté." Mr. Todhunter says of him, "The errors of D'Alembert are certainly surprising; they seem to me to indicate that he was little in the habit of enlarging his own views by comparing them with those of others. His criticisms of Clairaut prove that he had not really mastered the greatest work which had been written on the subject he was constantly studying. His readiness to publish unsound demonstrations and absolute errors is abundantly shown in the course of our criticism. On the whole the blunders revealed in the history of the 'Mathematical Theory of Probability,' and in the present history, constitute an extraordinary shade on a fame so bright as that of D'Alembert."

Here we must give an account of a work not mentioned in the History. The "Considerazioni sopra la Figura della Terra"[*] of Tommaso Narducci appeared about the year 1747. It comprises two Lemmas (in modern geometrical conics they would be for the ellipse (1), GN equal $\frac{CB^2\,CN}{CA^2}$, (2) radius of curvature equal $\frac{PG^3\,CA^2}{CB^4}$), and nine problems. The first problem is "Dati due gradi di meridiano e loro latidudine, trovare la ragione degl' assi, e gl' assi stessi;" the last is "Data la ragione de' due assi, che sia di 1 ad m, trovare nel meridiano un grado, che sia eguale al grado dell' equatore." It is an interesting piece of geometrical work.

In his § 490 Mr. Todhunter considers it curious that the (Cambridge) University library does not possess a complete copy of the famous work of Stay and Boscovich. His surprise will probably be increased when we state that, if we are not mistaken, neither do the libraries of the British Museum or that of the late Mr. Graves; at any rate, we do not remember to have met with Boscovich's commentary on the poem. "These writings furnish elementary accounts of the most important results which had been obtained up to their date, and reveal apparently great knowledge and judgment in Natural Philosophy." A copy of Boscovich's "Dissertatio de telluris figurâ, habita in seminario romano Soc. Jesu nunc primum aucta et illustrata ab ipsomet auctore, P. R. J. Boscovich," forms pp. 161-218 of vol. ii. of Giuliani's memoirs (cited at the foot, date 1744). In p. 184 he speaks of Maclaurin's "Fluxions" as "Newtono ipso dignissima;" there is a noteworthy passage, pp. 217, 218, and also a notice on p. xii.

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[*] Continued from p. 380.

[*] It occupies pp. 225-266 of vol. iii. of the "Memoria sopra la Fisica e Istoria Naturale di diversi valentuomini," edited by Carl Antonio Giuliani, in 4 vols. (1743-1747).

of the introduction of the volume.* The date on the copy of Frisi's dissertation (not seen by Mr. Todhunter) we consulted was 1751; it contains 8 pp. of *antecessio;* 86 pp. of text, 3 plates of figures, and 2 pp. of dedication. It consists of ten chapters, and may possibly, as Mr. Todhunter suggests, have been incorporated by Frisi in his "Cosmographia" (§ 532), though it certainly was not transferred bodily; on p. 65 he gets "axis terræ ad diametrum sibi normalem erit ut 229 : 230."

We have seen one of the works alluded to in § 551, viz., that of Bouguer; it is entitled, "Opérations faites par ordre de l'Académie . . . pour la vérification du dégré du méridien compris entre Paris et Amiens." Paris, 1757. 8vo. It consists of 28 pp., including two for title, and was read March 23, 1757.

In § 717 reference is made to the account of a measurement of an arc of the meridian in Lombardy by Beccaria, published at Turin "in 1744." We know of no work by Beccaria on this subject, except the "Gradus Taurinensis." The two copies of this which we have seen are dated 1774 and 1775; possibly from its position in the book the former of these dates is intended by Mr. Todhunter. It is a scarce work.

Mr. Todhunter has in § 725 devoted more attention to a supposed work by Newton than perhaps it really deserves. At any rate, De Morgan ("Budget of Paradoxes," p. 83) greatly doubts that Newton wrote it. He remarks that it has been treated with singular silence, and the name of the editor has never been given.

Lagrange contributes a memoir, in which he arrives by analysis at the point Maclaurin had reached by geometrical methods.

The operations carried on at Schehallien for ascertaining the density of the earth are next noticed, and with the conclusion of vol. i. the history of the two subjects during the century which followed the appearance of the "Principia" is nearly completed.

Some works mentioned as not having been seen (in § 738) may here be described.

The title of D'Anville's book is incorrectly given by Lalande.† For "*la* circonférence," read *sa;* for "*de l'équateur*," read *sur les parallèles*. There are 8 pp. of dedication (to Duc de Chartres), 20 pp. of *avertissement*, 3 pp. of *privilége*, &c, 11 pp. of observations, and 147 pp. of text, with a plate.

Lalande describes very accurately the contents of the "Anecdotes physiques et morales."

Mayer's "Basis Palatina" has 6 pp. of dedication, 14 pp. *lectori astronomo;* 23 pp. are taken up with "Series et ordo triangulorum quæ ex propriis suis observationibus, anno 1763 habitis, deduxit et correxit C. M.; 2 pp. of *conspectus totius operis*.

Hennert's work (Utrecht, 1778) contains five dissertations, of which the fourth is—"Sur le mouvement que prend un corps, quand il est parvenu au centre d'attraction, et sur l'attraction considerée comme principe universel." It takes up pp. 125-166. On p. 166 he says—

"Concluons de nos recherches qu'on n'a pas assès de preuves pour admettre l'attraction comme principe universel de tous les changemens qui arrivent dans le monde matériel. Nous avons vu que la cohésion des corps ne se déduit pas sans difficulté de l'attraction. . . . . Je sens bien, qu'il reste d'autres recherches à faire sur cette matière. J'en ai ebauché quelques unes que je pourrai publier si elles me paroissoient pouvoir contribuer au progrés des sciences physico-mathèmatiques."

His fifth dissertation is "Sur la figure de la terre relativement à la parallaxe de la lune et à la navigation." It occupies pp. 167-214. In § 18 we read, "Après avoir cherché inutilement de concilier les observations avec les hypothèses, dont les astronomes ont fait usage jusqu'aujourdhui, tachons de tirer un meilleur parti de notre formule générale," &c. And in § 25, "Il résulte de nos calcules que l'hypothèse de M. Bouguer et la notre donnent plus exactement le degré de longitude que l'hypothèse elliptique qui s'écarte considérablement de l'expérience." There are 4 pp. of dedication, 11 pp. of preface (interesting and amusing), 3 pp. of contents, 214 pp. of text, 2 pp. of errata, and 3 plates.

We may here describe a volume containing "Dissertationes de uniformitate motus diurni," by Hennert and Frisius, "præmio coronatæ," Petropoli, October 10, 1783. That by Hennert contains 42 pp., and *additamenta* making up 70 pp. in all; that by Frisius goes on to p. 112, and then 40 pp. more. They treat of attraction, but are not, apparently, of much importance.

Frisius it was who first introduced the ellipsoid as distinguished from the oblatum and the oblongum; from § 669, we learn, also, that he has no hesitation in adopting the truth as to the earth's motion.

Hube's work is in 87 pp. 8vo., with 1 page of plates. Its value may be seen from the following extract § 43:—

"Demonstravimus itaque tellurem nostram omnino esse homogeneam, vel potius densitatem variam illius partium, ratione universæ massæ, nullius fere momenti esse . . . probavimus porro, pendulorum experimentis, differentiam gravitatis sub polo et æquatore, tantam esse, ut terræ forma elliptica omnino esse nequeat . . . ad finem itaque meum pervenisse mihi videor, qui tantum fuit, ut experimentorum hucusque factorum ope, summâ qua fieri potuit evidentiâ, telluris formam certo definirem ac omnia dubia, quæ in hac re jure moveri possent, tollerem; neque adeo immorabor in consectariis variis nostræ theoriæ explicandis, quoniam vera telluris forma semel certo stabilita, haud difficile est, eo colligere atque perspicere, quæ inde necessario efficiuntur."

Further particulars concerning Thomas Williams (§ 988) are given in the "Budget of Paradoxes," p. 102.

The work referred to in § 1,000, is a 4to. volume, and contains 1 page of *avertissement*, 13 pp. of introduction, 1 page of table, 94 pp. of text, and 1 page of errata (in this Houslowheat is corrected to Hounslowheat). We think we have somewhere seen the preface ascribed to Legendre; it is clear, however, from the introduction (from the words "deux de mes confrères de l'Académie M.M. Mechain et Legendre, p. xiv.) that it is due to Cassini, as Lalande correctly states. With the copy we examined was bound up "Descriptions des moyens employés pour mesurer la base de Hounslow Heath dans la province de Middlesex, publiée dans le vol. lxxv. des 'Trans. Phil.' par le Major-Général William Roy, traduite de l'Anglais par M. de Prony," Paris, 1787. There is a *discours préliminaire du traducteur* 18 pp. There is a notice of 1 page, a page of errata: the translation occupies 80 pp., there are 3 tables, and 5 planches. Mr

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* We have also seen another work, the "De centro Gravitatis, editio altera, accedit disquisitio in centrum magnitudinis" (Rome, 1751), but it does not, if we remember, bear upon our subject.

† There is a very valuable copy of this work ("Bibliographie Astronomique") in the Graves' Library, containing Lalande's autograph notes, and many other interesting features.

Todhunter gives no account of this work: it is apparently a translation of the work described in § 984.

The following work by Delambre and Legendre (will this throw light on § 1,146?) does not seem to be described: it is "Methodes analytiques pour la détermination d'un arc du méridien précédées d'un memoire sur le même sujet," par A. M. Legendre. Paris, 1799. 4to.) In note ii. to "Methode pour determiner la longueur exacte du quart du meridien," par A. M. L. occurs *Legendre's Theorem.* There are 176 pp., 16 pp. of tables at the end, 5 pp. of "Observations sur quelques endroits du mêmoire du cit. Delambre," par A. M. Legendre; 2 plates, 7 pp. of *avertissement*, and 4 pp. of contents.

We now close our notes with a few remarks on §§ 531, 682, and 1,584. The memoir, which neither Playfair nor Mr. Todhunter have been able to procure a sight of, is to be met with in vol. i. of the Giuliani memoirs cited above, consequently its date is antecedent to 1743. It occupies pp. 65-88, and is entitled "Problema mecanicum de solido maximæ attractionis solutum, a P. Rogerio Josepho Boscovich." The problem is, "Data quantitate materiæ punctum attrahentis, in quacunque lege distantiarum invenire solidum ipsum continens, quod maximè omnium attrahat ipsum punctum positum in axe solidi producto ad datam distantiam ob ipsius solido vertice propiore." The author gives a geometrical and an analytical solution of the problem, and concludes, "Solutio geometrica in eo huic posteriori præstat quod ibi determinatur solidum maximæ attractionis etiam inter solida omnia irregularia, hic tantum inter solida genita rotatione curvæ circa axem."

The greater part of this second volume is taken up with the important writings of Laplace, Poisson, Gauss, Ivory, and Plana.

In the case of Laplace, who was, like some other writers, not in the habit of acknowledging his indebtedness to preceding authors, the result of the investigations is to restore his reputation to its proper eminence. "In the present history, and in that of Probability, I have gone over a third part of the collected mathematical works of Laplace; and to that extent the evidence of his great powers and achievements is, I hope, fully and fairly manifested."

Our work is now nearly done; were we to make use of all our notes, we might easily double and treble what we have written.* We have noted upwards of fifty articles which are interesting as sidelights: thus, § 227, Bouguer's remarks on what we now call the "personal equation;" § 710, transformation of variables in a triple integral, and many others. The value of the index would have been enhanced had reference been made to these; as it is, the index is much fuller than in the earlier volumes, and we have detected but few errors. This is no new feature, for all Mr. Todhunter's books are most carefully got up in this respect, and we have not met with a single important error in the mathematical work; the few mistakes we have come across are easily corrected.

There are indications here and there throughout the volumes that, should the writer be able to secure the requisite leisure, he will not want for subjects to exercise his special gifts upon. We sincerely hope that he will do so at an early date, and that it may be our lot to read the results. In the course of the preparation of the present work Mr. Todhunter has published, in the Royal and other Societies' proceedings, various papers which have grown out of his investigations in the history of our two subjects.*

R. TUCKER

TRAINING

Training in Theory and Practice. By Archibald Maclaren. Second Edition. (Macmillan & Co., 1874.)

WHEREVER we go, "The Boat Race" is the topic of the hour. Opinions are freely expressed as to the relative merits of the rival crews; and the risks and dangers incurred during the process of training, sturdily insisted on by some, are as obstinately denied by others. The respective values of the slow and quick stroke—the American fashion of "sliding-seats," and a variety of kindred questions, are eagerly debated, occasionally by men who really understand what they are talking about, but more frequently in order to make conversation, or, as the phrase goes, "for the sake of something to say."

In such a state of the public mind, it has by a happy chance been so ordered that the second edition of Mr. Maclaren's well-known work on Training should appear, at a moment when all will take unusual interest in its contents.

From a perusal of this treatise everyone may gather information calculated to be of service to him, not only for the time being but for the future. The man who likes to know a little of everthing, may with advantage indulge his curiosity about a subject which has perhaps hitherto been out of his line; while the father with a son in "the eight," and whose mind has been disturbed by letters in the papers, hinting at dire mischief as the outcome of training, may find substantial comfort in its pages. All those who, either from necessity or for amusement, undergo much physical exertion, will do well to imbue themselves with the author's teaching; and the "man of ease" or of sedentary occupation will find hints for health or sanitary lessons of a description which, if honestly carried out, would cause the ruin of half the doctors in London.

Mr. Maclaren uses "rowing" as the peg on which to hang his theme, and the reason he gives for so doing is as follows:—

> "It is the exercise most susceptible of being influenced by a judicious system of bodily preparation, being at once an act of considerable intricacy, demanding long and assiduous practice, and an exercise of considerable difficulty, involving the possession, although not in an equal degree, of both muscular and respiratory power, to promote which is the object of all training."

The question, what is training? and what is it meant to do? he answers thus:—"It is, to put the body with extreme and exceptional care under the influence of all the agents which promote its health and strength, in order to enable it to meet the extreme and exceptional demands upon its energies." There can be no doubt that the essence of this paragraph is contained in the words "extreme and exceptional care;"—for without such care,

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* In our remarks we have preferred to treat the subjects from an historical rather than from a mathematical point of view.

* Possibly some further memoirs of use might be found in the "Librorum in Bibliothecâ Speculæ Pulcovensis, anno 1858, exeunte contentorum catalogus systematicus," by Otto Struve. Petropoli, 1860.